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of the base mid way between the upper and lower bases ; and a =altitude of frustum. Then $\rho=\frac{1}{2}(R+r)$. $\therefore 4\rho^2=(R+r)^2$.

Volume of frustum= $\frac{1}{3}\pi a(R^2+r^2+Rr)=\frac{1}{3}\pi a(2R^2+2r^2+2Rr)=\frac{1}{3}\pi a[R^2+r^2+(R+r)^2]=\frac{1}{3}\pi a(R^2+r^2+4\rho^2)$.

The same method applies to frustums of pyramids, and all solids coming under the prismatoid formula as special cases.

Also solved in a more general manner by *B. F. SINE, P. S. BERG, J. SCHEFFER, HARVEY N. DAVIS, W. H. DRANE, and CHAS. C. CROSS.*

CALCULUS.

68. Proposed by *EDWARD DRAKE ROE, JR., A. M.*, Associate Professor of Mathematics, Oberlin College, Oberlin, Ohio.

If $a^{x^{\dots x}}$ to r steps be denoted by $a^{\overset{r}{x}}$, and if $y=x^{\overset{r}{x}}$, prove that

$$D_x y = x^{\overset{r}{x} + \overset{r-1}{x} + \dots + \overset{1}{x}} (\log x)^{r-1} (1 + \log x) + \sum_{k=2}^r x^{\overset{r}{x} + \overset{r-1}{x} + \dots + \overset{k-1}{x} - 1} (\log x)^{r-k}.$$

Solution by the PROPOSER.

If $y=f_1(x)^{f_2(x)}$, we obtain, by taking the logarithm of both sides of the equation, and differentiating, the formula

$$D_x y = f_1(x)^{f_2(x)} \log f_1(x) D_x f_2(x) + f_1(x)^{f_2(x)-1} f_2(x) D_x f_1(x).$$

If in this $f_1(x)=x$, $f_2(x)=x^{\overset{2}{x}}$, that is if $y=x^{\overset{2}{x}}$, we obtain

$$D_x y = x^{\overset{2}{x} + 1} \log x (1 + \log x) + x^{\overset{2}{x} + 1 - 1},$$

and the formula to be proved is true when $r=2$. It is evidently not true for values of $r < 2$. Assume that it is true for all other values of r

Let $y = x^{\overset{r+1}{x} = x^{\overset{r}{x}}}$. In the above formula put, $f_1(x)=x$, $f_2(x)=x^{\overset{r}{x}}$, and we get

$$D_x y = x^{\overset{r+1}{x}} \log x D_x x^{\overset{r}{x}} + x^{\overset{r+1}{x} - 1} \overset{r}{x},$$

but by this assumption this is

$$\begin{aligned} D_x y &= x^{\overset{r+1}{x}} \log x [x^{\overset{r}{x} + \overset{r-1}{x} + \dots + \overset{1}{x}} (\log x)^{r-1} (1 + \log x)] + x^{\overset{r+1}{x}} \log x \sum_{k=2}^r x^{\overset{r}{x} + \overset{r-1}{x} + \dots + \overset{k-1}{x} - 1} (\log x)^{r-k} \\ &\quad + x^{\overset{r+1}{x} + \overset{r}{x} - 1} \\ &= x^{\overset{r+1}{x} + \overset{r}{x} + \overset{r-1}{x} + \dots + \overset{1}{x}} (\log x)^r (1 + \log x) + \sum_{k=2}^{\overset{r+1}{x}} x^{\overset{r+1}{x} + \overset{r}{x} + \dots + \overset{k-1}{x} - 1} (\log x)^{\overset{r+1}{x} - k}. \end{aligned}$$

But this expression has the same form with respect to $r+1$, that the assumption had with respect to r , and since the assumption was true for $r=2$, it is also true for all values of r greater than 2, which is what we had to prove.

Erlangen, Bayern, Hauptstrasse 83II, 26 February, 1898.

Also solved by *C. W. M. BLACK, W. W. LANDIS, and G. B. M. ZERR.*